NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 6

Fall 2025

Problem 6

Find the smallest x such that $x^2 + xy = 312$ and $xy + y^2 = 264$. Justify your answer.

Solution. Add the left and right sides of the two equalities

$$\begin{aligned} x^2 + xy &= 312 \\ xy + y^2 &= 264 \end{aligned}$$

to obtain

$$\begin{aligned} & x^2 + 2xy + y^2 = 576 \\ \Rightarrow \quad & (x+y)^2 = 576 \\ \Rightarrow \quad & x + y = 24 \quad \text{or} \quad x + y = -24 \\ \Rightarrow \quad & y = 24 - x \quad \text{or} \quad y = -24 - x. \end{aligned}$$

Plugging these back into the first equality yields

$$x^2 + 24x - x^2 = 312 \quad \text{or} \quad x^2 - 24x - x^2 = 312.$$

Consequently,

$$x = \frac{312}{24} = 13 \quad \text{or} \quad x = -\frac{312}{24}.$$

Since we want the smallest such, the answer is $x = -13$.